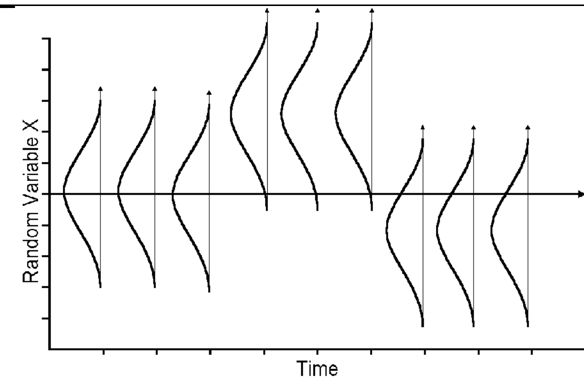
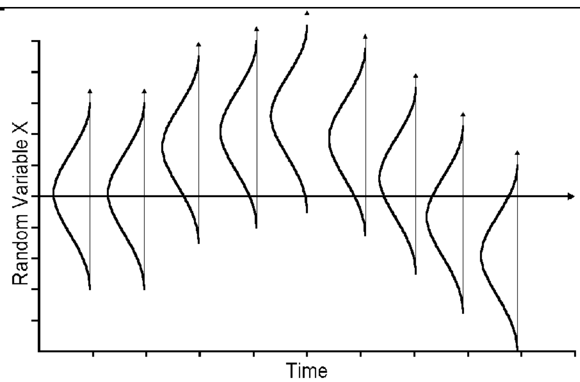
InIn4078: Statistical Quality Control
First exam - October 1999

Name: ____ **grade:** ____________

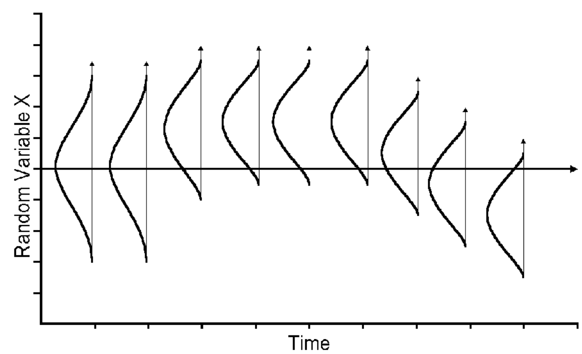
I. (20%) Consider the time-varying process behaviors shown in the following figures. If possible, match each of these patterns of process performance to a corresponding set of $\overline{X}$ and R charts shown on the following page.



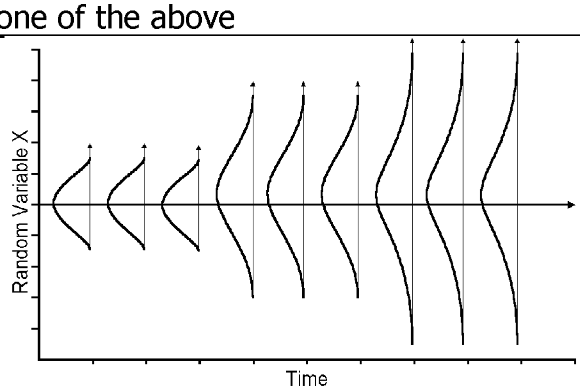
This process behavior is likely to correspond to
o) Control chart set I o) Control chart set II
o) Control chart set IV o) Control chart set VI
o) None of the above



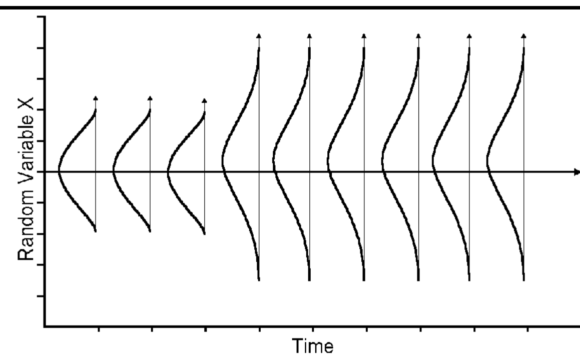
This process behavior is likely to correspond to
o) Control chart set I o) Control chart set II
o) Control chart set V o) Control chart set VI
o) None of the above



This process behavior is likely to correspond to
o) Control chart set I o) Control chart set III
o) Control chart set IV o) Control chart set VI
o) None of the above

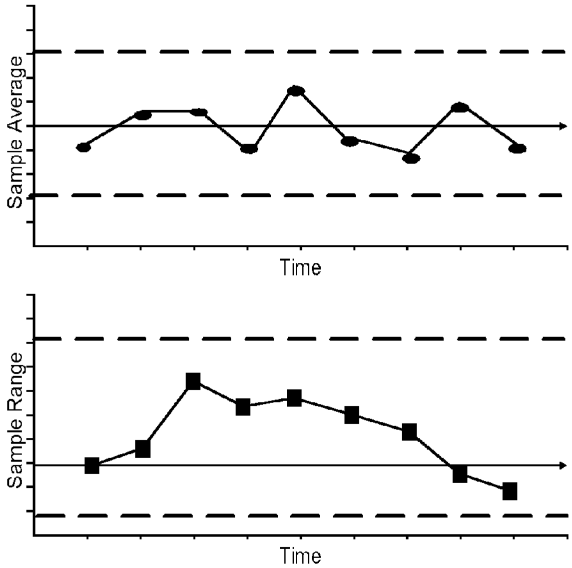


This process behavior is likely to correspond to
o) Control chart set I o) Control chart set II
o) Control chart set IV o) Control chart set VI
o) None of the above

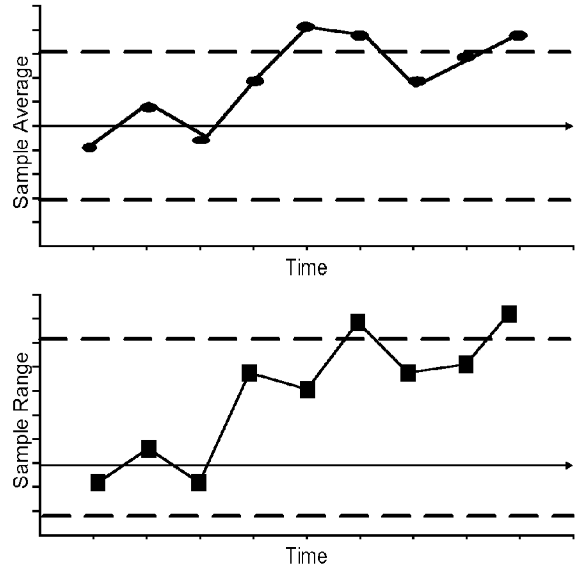


This process behavior is likely to correspond to
o) Control chart set I o) Control chart set III
o) Control chart set IV o) Control chart set V
o) None of the above

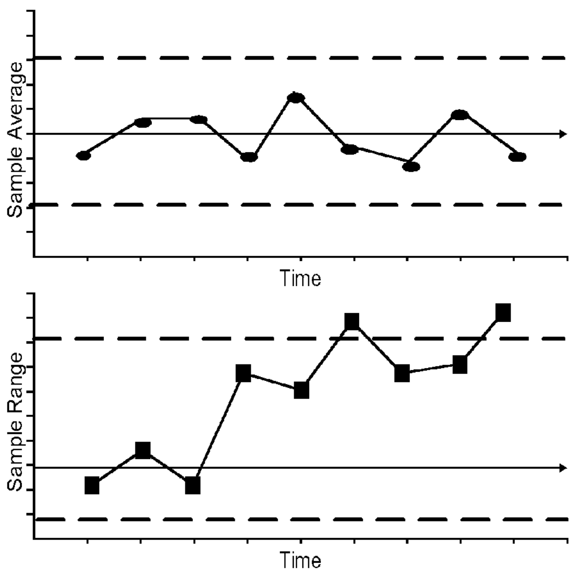
Control chart Set I



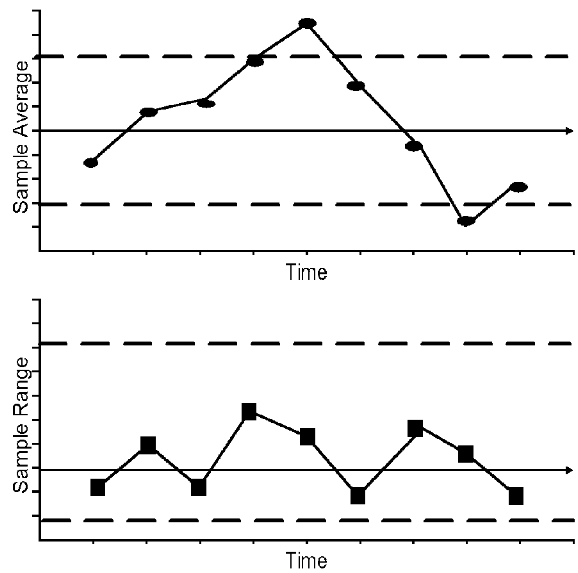
Control chart Set II



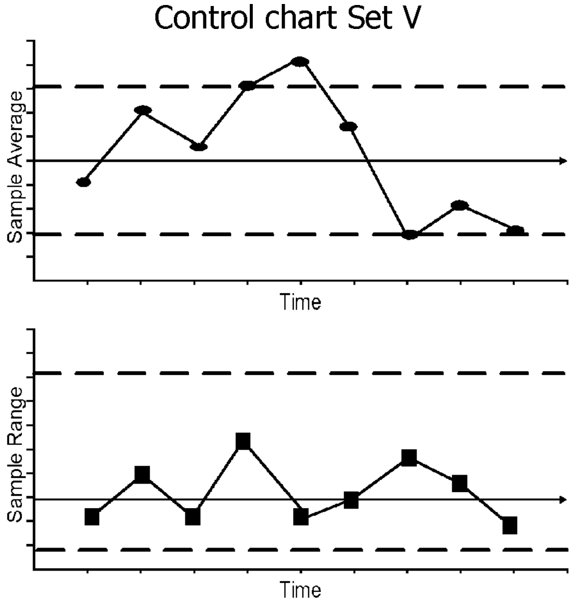
Control chart Set III



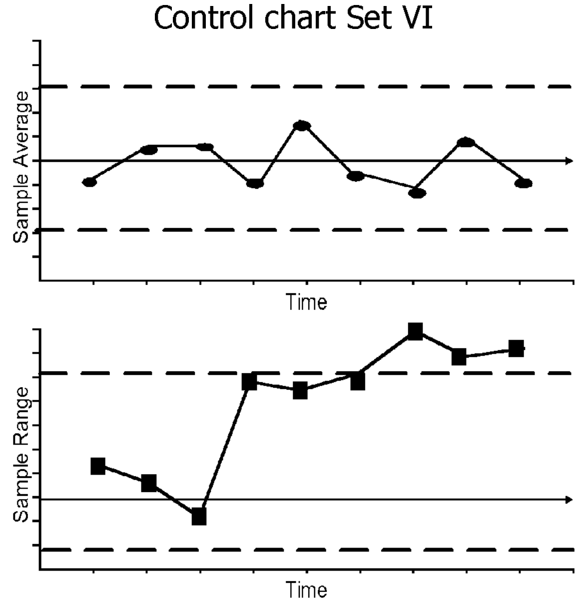
Control chart Set IV



Control chart Set V

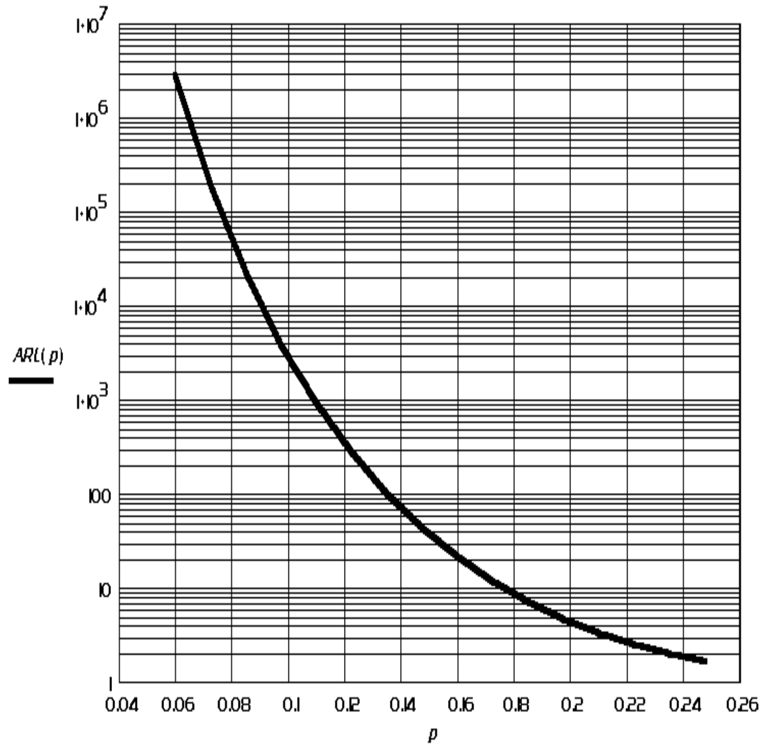


Control chart Set VI



II. (20%) For each of the following questions there are four possible answers. Select the best answer by checking (✔) the corresponding box.

The figure at the right shows the ARL curve for a typical chart to control the proportion defective. You might need to use the information provided by this plot to answer the following questions.



- When the true proportion defective changes to 16%, the expected number of samples to detect this change is approximately
  ❑ 11.2 samples
  ❑ 21 samples
  ❑ 21 n, where n = sample size
  ❑ It cannot be estimated with the given information.

- Assume that the in-control proportion defective used for designing this control chart was p_o = 0.12. Then
  ❑ The in-control ARL is 125 samples
  ❑ The probability of a Type I error for this chart is 0.29%
  ❑ The probability of a Type II error for this chart is 0.9971
  ❑ All of the above answers are false.

- If the **p** chart was designed so that the probability of a Type II error is 0.51603 when the proportion defective equals the upper control limit of the control chart, then
  ❑ UCL = 2.066% ❑ UCL = 23.2%
  ❑ UCL = 12.6% ❑ UCL cannot be estimated with the given information.

- The ARL curve of this typical **p** control chart suggests that
  ❑ Its LCL is greater than 0.06 ❑ Its UCL is greater than 0.26
  ❑ The control chart cannot detect changes in **p** to values greater than 0.26
  ❑ Its LCL is less than 6%

- Assume that the in-control proportion defective used for designing a typical **p** control chart is p_o = 0.13 and that UCL = 0.25, then the sample size used is
  ❑ n = 49 ❑ n = 71 ❑ n = 100 ❑ None of the above

III. (30%) Samples of 6 items are taken from a manufacturing process every hour. A normally distributed quality characteristic is measured and $\bar{X}$ and S values are calculated for each sample. After 50 samples have been analyzed, we have $\sum_{i=1}^{50} \bar{x}_i = 1100$ and $\sum_{i=1}^{50} S_i^2 = 1012.50$.

a. (6%) Assuming that the process is in statistical control, compute the control limits for the $\bar{X}$ and the S^2 control charts.

b. (6%) If σ is equal to the value that you estimated in (**a**), μ =24, and the specification limits for the quality characteristic are 22±10, what proportion of the items will be produced conforming to specifications?

c. (6%) If σ changes to 9.00, what is the probability that the S^2 chart (that you designed in **a**) detects this change in the next sample?

d. (6%) The person in charge of collecting the data and plotting $\bar{X}$ and S^2 is having problems calculating S^2 (he is a recent graduate from an infamous university); consequently, you have decided to control the process variability using an R chart instead of the S^2 chart. Compute the control limits for the R chart for samples of size 6.

e. (6%) Your supervisor has decided to change the sample size to 8 and to control the process using $\bar{X}$ and **S** charts. Compute the control limits for these new control charts.

IV. (30%) The in-control fraction nonconforming of an injection-molding operation is 5%.

a. (6%) If 75 items are inspected each shift, what are the control limits for the corresponding **p** chart?

b. (6%) With the control chart that you designed in (a), what is the probability of detecting a change to 10% in the fraction nonconforming on the first, second, or third sample after the change?

c. (6%) If the fraction nonconforming changes to 10%, what is the expected number of samples that one has to take to detect this change?

d. (6%) Assume that you want to continue using 3-σ limits, but you want to have a positive LCL. What is the smallest sample size that would yield a positive LCL? Recall that the in-control p = 3%.

e. (6%) How large should the sample size be if you wish the probability of detecting a shift (from the in-control p=5%) to p = 10% to be approximately 0.60?