Control charts for defects
Exam problem

In an optical-fiber manufacturing plant, it has been established that the **number of defects per 1.5 km** of fiber follows a Poisson distribution with an expected value of eight defects. If you are using a typical control chart, and 40 defects are found after inspecting 4.5 km of fiber; do you have enough evidence to declare the process out-of-control? Justify your answer.

Possible answers:

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a) To control the process, I will use a control chart for

Y = number of defects per 4.5 km of fiber

then, Y is Poisson and E[Y] = 8*(4.5/1.5) = 24 = Var[Y], and consequently,

UCL = $24 + 3\sqrt{24} = 38.69$; since in this case Y = 40 > UCL, we have evidence to declare the process **out of control**.

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b) To control the process, I will use a control chart for

U = number of defects per 1.0 km of fiber = Y/4.5

then, U is not a Poisson random variable, but

E[U] = E[Y/4.5] = 24/4.5 = 5.333, and Var[U] = Var[Y/4.5] = $24/(4.5)^2 = 1.185$

UCL = $5.333 + 3\sqrt{1.185} = 8.6$; since in this case U = 40/4.5 = 8.89 > UCL, we have evidence to declare the process **out of control**.

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c) To control the process, I will use a control chart for

V = number of defects per 1.5 km of fiber (in a sample of 4.5 km) = Y/3

then, V is not a Poisson random variable, but

E[V] = E[Y/3] = 8 and Var[V] = Var[Y/3] = 24/ 9 = 8 /3

UCL = $8 + 3\sqrt{(8/3)} = 12.89$; since in this case V = 40/3 = 13.33 > UCL, we have evidence to declare the process **out of control**.
